

Optimal PMU Placement Techniques: A Detailed Review and Case Study on the Indian Utility 62 Bus System

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Received: 15.10.2025 Accepted: 21.05.2026

Abstract – Optimal placement of Phasor Measurement Unit (PMU) is important for power system observability and reliable monitoring. The existing placement approaches emphasize full observability with minimum PMUs to achieve a cost-effective solution; however, the direct observability of critical buses and redundancy under contingency conditions has received limited attention. Consequently, minimum-PMU solutions may become vulnerable under PMU or transmission line outage conditions. In this paper, an attempt has been made to improve the system redundancy, security and reliability along with detailed, comprehensive review of existing Optimal PMU Placement (OPP) methods. A comparative analysis based on cost effectiveness, reliability, critical bus observability, system redundancy, and overall system security has also been carried out. A novel Depth of Redundancy (DOR) metric is introduced to quantify bus-level redundancy, and the System Observability Redundancy Index (SORI) is utilized to evaluate overall system redundancy. The framework is validated using IEEE 30 and 57 bus systems and implemented on the Indian Utility 62 bus system as a detailed case study. The results demonstrate that Integer Linear Programming (ILP) achieves cost-effective observability solution, whereas Depth First Search (DFS) achieves the highest observability redundancy and strong contingency resilience with a greater number of PMUs. Simulated Annealing (SA), however, achieves comparatively higher SORI and DOR values, thereby providing a balanced trade-off between cost-effectiveness, redundancy and monitoring. The study highlights that redundancy-oriented PMU placement significantly enhances monitoring reliability and contingency preparedness for practical utility-scale applications.

Keywords Phasor measurement units (PMUs), optimal PMU placement (OPP), system observability redundancy index (SORI), synchrophasor technology.

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1. Introduction

Phasor Measurement Units (PMUs) provide real-time, high-resolution, and time-synchronised voltage and current phasors across wide geographical regions, which are essential for power system observability. The concept of synchronized phasor measurement was first introduced in the 1980s, with advancements in GPS technology to achieve precise time synchronization [1]. This innovation allowed the measurement of voltage and current phasors accurately, which are critical for assessing the stability and reliability of power systems. The obtained time-stamped phasor data allows real-time monitoring of the power system, permitting detection, anticipation, and correction of issues during irregular conditions. Hence, PMUs are widely deployed across the power system for post-disturbance event analysis, wide-area monitoring, real-time state estimation, and enhancement of grid resilience [2]. Recent developments in smart grid and energy management systems further highlight the growing importance of synchronized measurements, real-time monitoring, and intelligent control strategies. IoT-enabled monitoring frameworks have been applied in electric vehicle charging systems for real-time parameter acquisition and operational management, while advanced GPS-assisted control strategies have been investigated for hybrid electric vehicle applications [3],[4]. Similarly, the increasing penetration of renewable energy resources,

particularly photovoltaic (PV) systems, has introduced new operational challenges associated with voltage regulation and system stability in distribution networks [5]. To address these challenges, recent studies have explored intelligent data-driven approaches, including hybrid graph neural network models, for overvoltage prediction and monitoring in PV-dominated grids [6]. Furthermore, reliable monitoring and observability are particularly important in remote and islanded microgrids operating under extreme conditions, where resilient system operation becomes essential [7]. These advancements collectively emphasize the increasing significance of accurate synchronized measurements and intelligent monitoring frameworks in modern power systems.

Although PMUs enhance power system monitoring, their high cost and need for communication infrastructure limit deployment at every bus. Moreover, it is not mandatory to place a PMU at every bus, as the voltage phasor at a bus connected to a PMU-equipped bus can be calculated using branch parameters and the measured current phasor from the connecting branch [8]. Typically, a minimum of 20%–30% of the total number of system buses is deemed sufficient for PMU deployment [9]. Therefore, the strategic PMU placement, considering factors such as installation cost [1], measurement redundancy [10], and fault detection capabilities [11], has become a focal point of research. Several optimization techniques have been proposed to solve the OPP problem. These are categorized as deterministic and metaheuristic algorithms. These approaches give a different number of PMUs with different locations on the same system. Hence, to identify the best method or location, several indices have been developed, including the ‘System Observability Redundancy Index’ (SORI) [5]. The high value of SORI represents high observability. Also, to lower the number of PMUs, the effect of ‘Zero Injection Buses’ (ZIBs) has been considered by various researchers [12].

Weak and generator buses are more prone to voltage collapse, threatening system security [13]. However, the direct observability of such critical buses has received comparatively less attention in existing OPP studies. Therefore, this work presents a comprehensive review of OPP techniques and evaluates PMU placement strategies based on observability, redundancy, and direct monitoring capability of weak and generator buses. The main contributions of this paper are as follows:

1. A comprehensive review of existing techniques, including a description, the research focus, and the bus system used for OPP, has been conducted. The results reported in the literature using existing techniques have been analysed to determine the best location based on SORI values.
2. A new Indian Utility 62bus system has been designed in MATLAB, providing practical insights by taking data from [14].
3. PMU locations have been obtained on the IEEE 30 and IEEE 57 for validation and implemented on the Indian

Utility 62 bus systems. Using the integer linear programming (ILP), Depth-first search (DFS), Simulated Annealing (SA), and Recursive Spanning Tree Search (RSTS), considering the ZIB effect. Multiple solutions for PMU locations have been obtained using different techniques.

4. To demonstrate the effectiveness of the multiple solutions, SORI and a new proposed metric, Depth of Redundancy (DOR), have been calculated.
5. A comparative analysis based on cost effectiveness, reliability, critical bus observability, system redundancy, and overall system security has also been carried out.

The proposed approach provides a more reliable framework for redundancy-aware PMU placement compared to conventional observability-based methods. The fundamentals of phasor technology, including its role in synchronized measurements, wide-area monitoring, and real-time grid applications, have been explored in Section 2. Section 3 presents various OPP algorithms. In Section 4, comprehensive reviews of existing OPP techniques, analyzing their efficiency, computational complexity, and adaptability to different power system configurations, have been presented. These approaches are validated in Section 5, presenting simulation results, comparing various methods based on observability indices, redundancy, and computational efficiency. Finally, section 6 concludes with key findings, limitations, and future research directions for improving PMU deployment strategies.

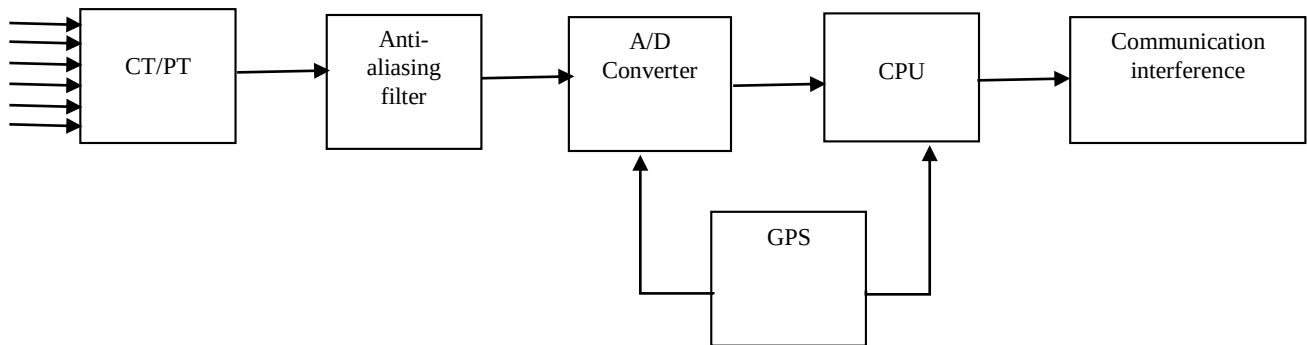


Fig. 1. Block diagram of phasor assisted device.

2.2. Optimal PMU Placement (OPP)

The deployment of PMUs is essential in power systems as they provide time-stamped, synchronized phasor measurements. When all bus voltage and current phasors are known, then the system is said to be fully observable. However, due to the elevated cost and the requirement of communication facilities make it impossible to place PMU at each bus. Hence, OPP aims to reduce the number of PMUs placed in the system while ensuring complete observability. This approach seeks to strike a balance between effective system monitoring and minimizing the associated costs and

2. Phasor Technology

Phasor technology, also known as phasor measurement technology, plays a critical role in modern power system monitoring and control. The complex equivalent of a sinusoidal waveform is defined as a phasor, where the modulus represents the amplitude and the phase is denoted by the angle of the signal. This mathematical representation allows for simplified analysis of steady-state AC electrical signals. PMUs, the core component of this technology, are explained in the subsection below [15].

2.1. Phasor Measurement Unit

PMUs provide high-speed, time-synchronized measurements of voltage and current phasors across a power system by placing on some locations. The voltage and current phasor of a bus connected with a PMU-equipped bus can be calculated using branch parameters, and the measured current phasor at a bus connected to a PMU-equipped bus can be calculated using branches. By using GPS-based synchronization, PMUs enable real-time visibility into the dynamic behaviour of the system, capturing both the magnitude and phase angle of electrical quantities with much higher resolution and speed compared to traditional SCADA systems. This level of insight is crucial for detecting voltage instabilities, identifying system oscillations, and implementing wide-area control strategies.

resource requirements. The objective is to observe the power system with the minimum number of PMUs by strategically defining their positions [9].

2.3. Problem Formulation

The typical observability and OPP in power systems involves determining the minimum number of PMUs required. n_{pmu} , and their optimal locations $S(n_{pmu})$. This model can be extended to include other constraints like ZIBs, a limited number of channels, etc. [7]:

$$\min_{n_{pmu}} \min_{S(n_{pmu})} G(n_{pmu}, S(n_{pmu})) \quad (1)$$

$$\text{Subjected to: } f(n_{pmu}, S(n_{pmu})) = 1 \quad (2)$$

where:

$G(n_{pmu}, S(n_{pmu}))$: the number of unobservable buses

$S(n_{pmu})$: a set of buses where PMUs are installed

$f(n_{pmu}, S(n_{pmu}))$: logical function ensuring system observability and constraint satisfaction

The logical function $f(\cdot)$ is defined as:

$$f(\cdot) = O_{bs}(n_{pmu}, S(n_{pmu})) \cdot C_{on}(n_{pmu}, S(n_{pmu})) \quad (3)$$

where:

$O_{bs}(\cdot)$: observability function (equals 1 if the system is fully observable, otherwise 0)

$C_{on}(\cdot)$: constraint evaluation function (accounts for practical constraints such as Zero Injection Buses (ZIBs), channel limits, etc.).

‘+’ and ‘·’ denote the logical OR and AND operators, respectively.

There is no unique solution for the PMU placement problem. This is because there are different sets of constraints taken by different researchers according to their objectives for finding solutions with different techniques. A ZIB is a bus within a power system that is neither connected to a load nor a generator. As a result, the installation of a PMU at every bus is not required to achieve system observability. The presence of ZIBs can effectively reduce the number of PMUs needed to make the power system completely observable [16]. This constraint has also been added for PMU placement.

2.4. Performance indices

This paper introduces two performance metrics to evaluate the robustness and effectiveness of PMU placement:

➤ System Observability Redundancy Index (SORI)

The SORI quantifies the extent to which strategically placed PMUs can monitor the entire power system [16]. A higher SORI indicates a more reliable PMU-based monitoring system. In the context of the paper, SORI is employed to identify the algorithm that provides the maximum measurement redundancy, enhancing the robustness and reliability of the monitoring system. The SORI is computed using the following equation:

$$SORI = \sum_{j=1}^N \sum_{i=1}^N A_{ij} X_i \quad (4)$$

A_{ij} : is a binary connectivity matrix

X_i : is the optimal location of PMUs for complete observability.

N = total number of buses

The maximum value of SORI means that a larger portion of the system will remain observable in case of a PMU outage.

➤ Depth of Redundancy (DOR)

DOR quantifies the number of independent measurement paths (PMU paths) through which each bus is observable. A higher DOR implies a more resilient configuration capable of tolerating PMU failures or communication disruptions [7].

Mathematically, for bus i ,

$$DOR(i) = \sum_{j=1}^{N_{PMU}} \delta_{ij} \quad (5)$$

where:

$\delta_{ij} = 1$, if bus i is observable by PMU j (either directly or indirectly), otherwise $\delta_{ij} = 0$

This metric represents the number of independent PMU measurement paths through which bus i is observed.

N_{PMU} = the total number of placed PMUs

$DOR(i)$ thus represents the number of independent measurement paths covering bus i . A bus with $DOR_i = 1$ is critically observable — its loss of observability is guaranteed if its sole covering PMU fails.

The average DOR across all buses serves as a system-level redundancy score.

The average DOR is given by:

$$DOR_{avg} = \frac{1}{N} \sum_{i=1}^N DOR_i \quad (6)$$

Unlike SORI, DOR explicitly captures redundancy depth at each bus, making it suitable for identifying weak buses.

➤ Mathematical Relationship between SORI and DOR

SORI is calculated by adding the DOR values of all buses in the system and can be expressed as:

$$SORI = \sum_{i=1}^N DOR_i = N \times DOR_{avg} \quad (7)$$

where:

DOR_i represents the redundancy level of the i^{th} bus,

DOR_{avg} is the average Depth of Redundancy of the system,

N is the total number of buses.

This relation shows that SORI and average DOR are directly connected mathematically. However, both metrics provide different types of information. SORI gives only the overall redundancy level of the complete network, whereas DOR provides bus-wise redundancy information. Therefore, two systems may have the same SORI value but completely different redundancy distributions across buses.

For example, a system with $SORI = 79$ may have some buses observed through multiple PMUs while other buses are observed through only a single PMU. In such cases, the overall redundancy appears high, but certain buses remain vulnerable. DOR helps identify these weak points by showing the redundancy available at each bus. Buses with $DOR = 1$ are considered critical because the loss of a single PMU may make them unobservable. This is especially important for generator buses and weak buses, where reliable monitoring is essential for secure system operation. The DOR values considered in this work are based on the network topology, which remains unchanged under normal load variations. Therefore, load changes do not affect DOR unless a topological change, such as a transmission line outage, occurs.

3. OPP Algorithms

This section provides an overview of different approaches for solving the OPP problem. It includes both mathematical and heuristic approaches.

3.1. Integer Linear Programming (ILP)

ILP is a deterministic optimization technique used to find the OPP problem. The objective is to minimize the number of PMUs with maximum observability and hence minimize total cost.

Formulation problem [12] :

$$\text{Min } \sum_{i=1}^N C_i X_i \quad (8)$$

$$\text{Subjected to : } AX \geq 1 \quad (9)$$

where:

N is the total number of buses

C_i : installation cost of PMU at bus i

X_i : binary decision variable, defined as

$$X_i = \begin{cases} 1 & \text{if PMU is installed} \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

A is a binary connectivity matrix defined as

$$A_{ij} = \begin{cases} 1 & \text{if } i = j \text{ or } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

The constraint $Ax \geq 1$ ensures that each bus is observable either directly (through a PMU installed at that bus) or indirectly (through adjacent buses). The problem of OPP is solved using the above procedure in MATLAB.

3.2. Quadratic Programming (QP)

QP serves as a fundamental tool in addressing non-linear programming, notably utilized in optimizing linearly constrained quadratic objective functions. Recent research, as exemplified by [17], employs linear QP for OPP, aiming firstly to attain a complete topologically observable system under normal operation or in cases of PMU or line outage, and secondly, to maximize measurement redundancy. Additionally, [18] utilizes ‘Integer Quadratic Programming’ for PMU placement, ensuring observability. Uncertainty evaluation within the system is addressed through various methods such as classical uncertainty propagation theory, Monte Carlo simulations, and Random Fuzzy Variable (RFVs), as explored in different studies. Furthermore, advancements include the extension of OPP to the ‘Optimal PMU-Communication link Placement’ (OPLP) problem, introduced by [19], where mixed-integer quadratically constrained programming is applied, considering PMU and communication link capacities.

3.3. Simulated Annealing (SA)

SA is a probabilistic optimization technique motivated by the annealing process in metallurgy, where an initial random configuration of PMUs is iteratively modified to find optimal

placement by evaluating solutions based on reduced PMUs and improved observability. By gradually reducing the acceptance probability of worse solutions as the temperature decreases, SA escapes local optima and refines solutions until the stopping criterion is met, achieving near optimal or optimal PMU placements [1]. A practical solution for the communication-constrained PMU placement problem is the use of the SA method, which addresses the OPP problem by managing incomplete observability [20]. The case studies are simulated within the ‘Power System Analysis Toolbox’ (PSAT) using MATLAB software.

3.4. Depth First Search (DFS)

The DFS method is a stochastic optimization technique. In this algorithm, the initial step involves exploring the node connected to the highest number of branches within the network. Subsequently, a PMU is strategically placed on this bus. In cases where two or more nodes exhibit an equally substantial number of branches, the algorithm randomly selects nodes for the placement of PMUs. This iterative operation is systematically applied to all nodes throughout the network, ensuring comprehensive observability of the entire system [16], [21]. Initially, the case studies are simulated within the PSAT using MATLAB software. Subsequently, power flow analysis is executed to determine the power flows through the Newton-Raphson method. At last, the optimal placement is found utilizing the PMU placement toolbox within PSAT, employing the DFS.

3.5. Recursive Spanning Tree Search (RSTS)

RSTS is an algorithm that dynamically finds the best way from source to destination within a graph while avoiding bridge loops that could lead to inaccurate results. This technique involves the recursive exploration of multiple spanning trees, each starting from a different node (bus). In power systems, RSTS is particularly valuable for OPP problems. In [22] spanning trees use graphs to find the OPP based on the notion of depth of unobservability. This approach makes sure that the distance between unobserved and observed buses is minimized and that PMUs are distributed uniformly across the network, enhancing the efficiency and reliability of the monitoring system.

3.6. Tabu Search

Tabu search develops as a promising metaheuristic approach for optimizing PMU placement in power systems, aiming to minimize the number of PMUs while ensuring full observability and redundancy. Unlike conventional heuristic methods, this method utilizes a flexible memory mechanism to avoid revisiting previously explored solutions, thereby enhancing exploration efficiency.

Previous works have demonstrated the effectiveness of Tabu search in addressing the OPP problem, heuristic rules and prioritized lists to guide the search process towards optimal solutions [23]. Additionally, the recursive Tabu search method

(RTS) proposed in the literature introduces an iterative approach, where multiple executions of conventional Tabu search are performed with each initialization guided by the best solution obtained from prior executions, further enhancing the algorithm's convergence and solution quality [24].

3.7. Particle Swarm Optimization (PSO)

PSO harnesses swarm intelligence, mimicking the behaviour of natural phenomena like bird flocking or fish schooling, to iteratively optimize solutions. Specifically, Binary PSO (BPSO) has been introduced for discrete problem solving, proving effective for OPP. This approach, as demonstrated by recent research, aims to minimize the number of PMU installations while ensuring complete system observability and various constraints, such as a limited number of PMUs, critical bus measurements, the effect of ZIBs, PMU outages, line outages and contingency analysis have been included as an objective function in different PSO algorithms [25].

4. Review of OPP Techniques

The literature review for the OPP problem systematically describes diverse studies, methodologies, and outcomes related to the strategic locations and count of PMUs in power systems. Table 1 provides an overview of metaheuristic algorithms employed in studies, while Table 2 focuses on deterministic algorithms. The literature review indicates that most researchers aim to minimize the number of PMUs while optimizing their locations for maximum redundancy. The different solution has been obtained by researchers, considering different constraints such as zero injection buses, depth of unobservability, line and PMU outages, and fixed or variable channels for PMUs. Various techniques, ranging from single to multi-objective optimization, have been employed to address the OPP problem, emphasizing achieving complete system observability with a minimal number of PMUs and maximizing measurement redundancy simultaneously. Hence, obtaining the optimal position of PMUs involves addressing practical constraints such as zero injection effect, PMU outages, line outages, measurement redundancy, incorporating traditional measurements, accounting for existing PMUs,

Table1. Metaheuristic algorithms

Ref.	Method Adopted/ Bus system used	Description/Features	Focus
[23]	Tabu search algorithm (TSA)/ IEEE 14, 57, and NE 39 bus systems.	Utilizes an augmented incidence matrix to achieve full system observability with reduced computation time.	Minimize PMUs for full observability.
[24]	Recursive Tabu Search (RTS) / IEEE14, 30, 57, 118, New England (NE) 39, 2383 Bus system	Applies Tabu Search recursively, using the best solutions from previous runs; combines numerical observability with metaheuristics.	Minimize PMUs for full observability.
[25]	Binary search algorithm/ IEEE14,24,30, and NE 298 Bus system	An exhaustive search approach has been used to determine the minimum number and OPP for state estimation, considering single branch outages.	Global optimum with branch outage.
[26]	Modified Binary Particle Swarm Optimization (BPSO) / IEEE 14, 30, 39, 57, and 118 Bus system	Considers ZIBs, PMU loss, and line outages; introduces new topological rules for observability analysis.	Robust OPP considering ZIBs.
[27]	Multi-Stage Simulated Annealing (MSSA) / IEEE14, 30 Bus System.	Integrates PMUs with conventional meters using a spanning measurement subgraph approach.	OPP with conventional measurements.
[28]	Exhaustive Binary Search Algorithm (EBSA)/ IEEE 9, 14, 24, 30 Bus system	Reduces search space via connectivity and symmetry constraints; validated via Monte Carlo simulations.	Reduces search space and computation time with full observability.
[29]	Graph-based method / IEEE14, 30, 57, and 118 Bus system.	Provides a GUI-based tool for interactive system modelling and PMU placement with SORI.	OPP with visualization and SORI maximization
[30]	Genetic Algorithm (GA)/ IEEE30, 57, 118, and 114 Algerian Bus system.	Uses evolutionary optimization to minimize cost while achieving observability.	Cost-efficient OPP
[31]	Immunity Genetic Algorithm (IGA)/ IEEE14, 30, 39, 57, and 118 Bus system.	Improves upon GA by avoiding premature convergence using immune operators.	Faster convergence for full observability
[32]	Non-dominated Sorting Genetic Algorithm /IEEE 39 and 118 Bus system.	Multi-objective optimization balancing minimal PMUs and high redundancy; uses the Pareto front.	OPP with maximum measurement redundancy.

[33]	Non-dominated Sorting Differential Evolution (NSDE)/ IEEE 14 and 39 Bus system.	Multi-objective model to minimize PMUs and ensure N-1 measurement reliability.	OPP with reliability optimization
[34]	Genetic Algorithm-Based Approach/ IEEE 14 and NE 39 Bus System	Considers contingency scenarios using a Radial Basis Function network post GA-based placement.	Contingency-aware OPP.
[35]	Improved cuckoo and PSO Method / IEEE 14, 24, 30, and 57 Bus system	Evaluates radial systems and ZIBs; Cuckoo performs better in iterations.	OPP with radial topology and ZIBs.
[36]	BPSO with Integrated Mutation / IEEE 300 Bus system.	Addresses PMU channel limits, ZIBs, and PMU loss to reduce computational load.	Scalable OPP with constraints
[37]	Weighted Least Squares (WLS) - based OPP/ IEEE14, 30, 118, and China 211 Bus system.	Smart grid-oriented; considers ZIBs, channel limits, PMU failures, and state estimation errors.	OPP for smart grid & state estimation
[38]	Fuzzy Logic and Discrete Artificial Bee Colony (FL-DABC)/ IEEE14, 30, 57 Bus system	Suitable for microgrid systems; accounts for ZIBs, PMU loss, and line outages.	OPP under contingency in microgrids.

Table 2. Deterministic algorithms

Ref.	Method Adopted/ Bus system used	Description/Features	Focus
[39]	Integer Linear Programming (ILP) based algorithm / IEEE 14, NE 39, and Northern Region Power Grid (NRPG) 246 Bus systems	Incorporates voltage stability-based contingency scenarios into ILP to identify critical PMU placements in large networks.	Contingency-based OPP
[7]	ILP/ IEEE 14, 57, and 118 bus system.	Considers ZIBs and uses the Bus Observability Index (BOI) and SORI to rank optimal placements.	OPP with maximum observability.
[40]	Sequential Quadratic Programming (SQP) Algorithm/ IEEE 14, 24, 30, and NE-39 Bus system	Provides multiple OPP configurations while considering PMU channel limits. Compared with the BILP and MILP results.	OPP with channel constraints
[41]	Binary Integer Linear Programming (BILP)/ IEEE- 7, 14, 30, 39, 57, and 118 Bus systems.	Uses bus and line connectivity matrices, addressing PMU channel limits to minimize the total PMUs needed.	Minimized PMU count with connectivity matrix.
[42]	BILP/ IEEE 14, IEEE 30, and 57 Bus system	Generalized ILP approach supporting redundant and incomplete observability scenarios. Compares outcomes with and without ZIB consideration.	Comparison of ZIB impact
[16]	MILP/ Nairobi Region 30-bus system.	Combines MILP with Artificial Bee Colony (ABC) and Depth-First search to optimize PMU placement in Kenya's power grid. Evaluated using SORI.	OPP with maximum redundancy.
[43]	MILP/ NE 10-unit 39 and IEEE 118-Bus system	Formulates MILP with backbone-resilience planning. The observability index is applied to evaluate the impact on both the backbone and the full network.	OPP with ZIBs and resilience planning.

considering communication limits, and addressing weakly observable buses.

It is seen in literature that the OPP problem lacks a unique solution. Hence, to identify the best location, a key index, SORI, has been utilized [16]. Each optimal location has its own SORI, and the maximum SORI corresponding to the PMU location presents the maximum measurement redundancy.

Therefore, a greater SORI value signifies a more reliable monitoring system.

5. Simulations and Results

The OPP techniques ILP, DFS, SA, and RSTS have been validated on IEEE 30-bus and IEEE 57-bus systems, and

implemented on the Indian Utility 62-bus system for a case study. The system models are developed using MATPOWER to ensure accurate network representation and observability analysis.

Since SA and RSTS are stochastic optimization methods employing probabilistic search mechanisms and random initialization, their obtained PMU placement configurations may vary across independent executions. Therefore, statistical analysis was performed using 30 independent runs with different random seeds to evaluate convergence consistency and solution robustness. The best 6 solutions based on PMU count and maximum SORI have been considered for analysis, as shown in Table 3.

In contrast, ILP and DFS are deterministic optimization approaches that produce identical solutions for repeated executions under the same network conditions. The DFS method implemented in this work follows a fixed in-degree-based search strategy without random exploration or probabilistic acceptance criteria.

The SORI is calculated for each optimal PMU configuration, and the newly introduced DOR metric is computed to quantify direct observability of voltage critical buses, particularly at generator buses and identified weak buses [15], [44].

5.1. Case study I: IEEE 30-Bus system

Figure 2 presents the PMU requirements and corresponding SORI values for the IEEE 30-bus system across different optimisation methods. The results show minimal variation in PMU requirements, with both ILP and RSTS achieving complete system observability using the minimum number of PMUs (7). ILP achieves superior performance, with a SORI of 35, compared with RSTS, which has a SORI of 31, indicating better redundancy distribution despite the identical PMU count. As shown in Table 3, SA requires one additional PMU (8) but achieves an improved SORI of 39. DFS deploys the maximum PMUs (10), achieving the highest SORI of 45, providing better redundancy than ILP but at a high cost.

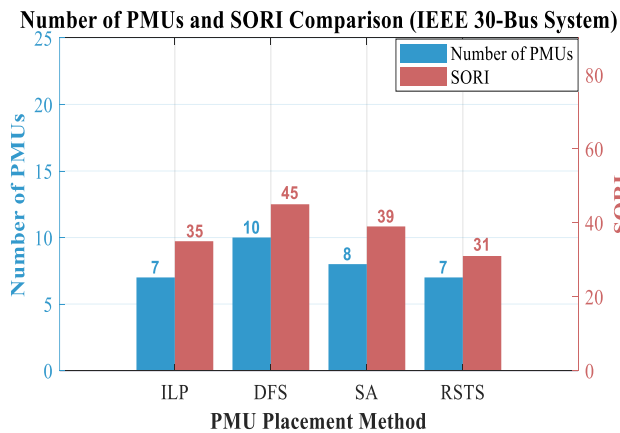


Fig. 2. Number of PMUs obtained for the IEEE 30 bus system using various techniques with their SORI.

5.2. Case Study II: IEEE 57-Bus system

Figure 3 presents the performance comparison of various methods of OPP for the IEEE 57-bus system, significantly showing variations in both PMU counts and SORI. It can be observed that ILP achieves observability with 11 PMUs and SORI of 54, representing the most economical solution. RSTS requires a marginally higher PMU count (12) but achieves lower SORI (51), indicating suboptimal redundancy distribution. SA provides a balanced approach with 14 PMUs and an SORI of 59. DFS demonstrates maximum redundancy capability with SORI of 85 using 22 PMUs, achieving higher redundancy requiring more PMUs, making it economically challenging for practical implementation.

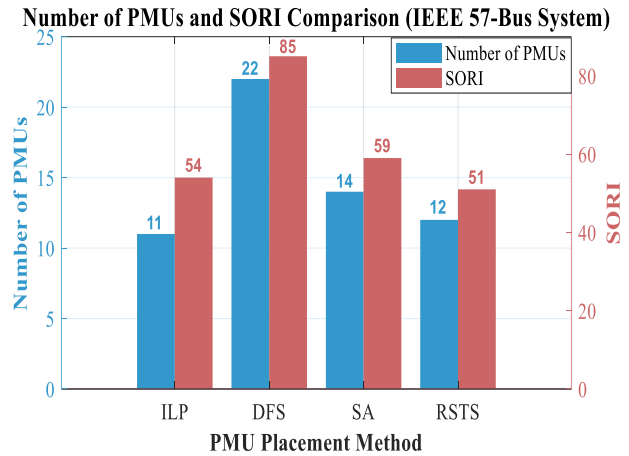


Fig. 3. Number of PMUs obtained for the IEEE 57 bus system using various techniques with their SORI.

5.3. Case Study III: Indian Utility 62 Bus System

The Indian Utility 62-bus system is the selected network to evaluate the effectiveness, robustness, and practical applicability of the proposed PMU placement methodologies under realistic operating conditions. Compared with standard IEEE test systems, the Indian Utility 62-bus system features a larger, more complex topology with multiple generation buses, interconnected corridors, and varying observability conditions. Therefore, this case study is used to assess not only optimal PMU placement performance but also resilience under contingency conditions.

5.3.1. System Description and PMU Placement Results

The Indian Utility 62-bus system is a real-world meshed transmission network representative of a regional grid in India. The system comprises 62 buses, 89 transmission branches, 11 transformers, 32 load buses, and 19 generating units distributed across 8 distinct generator buses (1, 14, 17, 25, 32, 37, 51, and 58). Multiple generating units are connected to common buses; therefore, PMU placement analysis focuses on generator buses rather than individual generating units. The network operates at three voltage levels: 220 kV, 132 kV, and 66 kV, interconnected through step-down transformers. Bus 1 serves as the slack (reference) bus. The system data, including bus admittance matrix, line parameters, weak buses (11, 20, 30 and 31) and

loading conditions, are available in [45]. Figure 5 presents the single-line diagram of the Indian Utility 62-bus system with optimal PMU placement locations. The diagram clearly illustrates the strategic positioning of PMUs to achieve comprehensive system observability while prioritizing critical generator and weak bus monitoring.

5.3.2. Multi-Objective Performance Comparison

The results obtained for the Indian Utility 62- Bus system in Table 3 have been presented in Figure 4, showing the variation in PMU count and corresponding SORI values obtained from multiple optimization runs of different OPP methods. The solution diversity is observed in SA and RSTS compared to ILP and DFS. The multiple configurations obtained from the SA and RSTS methods highlight the stochastic nature of these algorithms. SA generates solutions ranging from 15-17 PMUs with SORI values 69-79, while RSTS produces consistent solutions with 15 PMUs and SORI values ranging from 68-71. These findings emphasize that higher SORI alone does not guarantee fault tolerance. Average DOR and DOR of critical buses are better indicators of robustness in real-world grid monitoring. Furthermore, since multiple feasible solutions were obtained from the SA and RSTS methods, the PMU configurations corresponding to the highest SORI values were selected for further investigation. A comprehensive performance analysis was then carried out, considering not only the overall observability redundancy but also the direct observability of critical buses, including generator and weak buses.

Number of PMU Placement Methods Comparison (Indian Utility 62-Bus System)

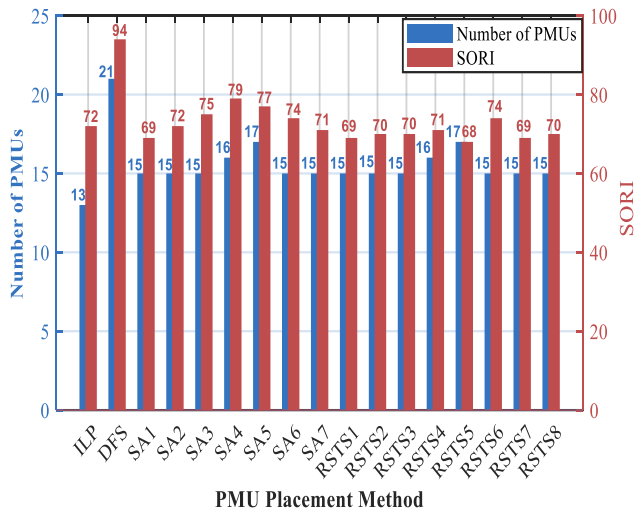


Fig. 4. Number of PMUs obtained for the Indian Utility 62-Bus System using various techniques with their SORI.

Table 4 presents the detailed comparison of all methods in terms of average DOR, SORI, generator bus observability, and weak bus coverage. The analysis identifies SA as the most balanced solution, achieving 100% generator bus observability with a 9.7% improvement in SORI compared to ILP, while requiring only 23.1% additional PMUs. This demonstrates that SA provides an effective trade-off between redundancy

enhancement and implementation cost for reliable real-world grid monitoring.

The critical bus coverage analysis in Table 5 further highlights the effectiveness of the SA-based PMU placement strategy. Although all methods achieved complete system observability, major differences were observed in critical bus monitoring performance. SA achieved 100% generator bus coverage and complete weak bus observability, demonstrating superior monitoring capability for strategically important buses. In contrast, ILP and RSTS observed only 62.5% of generator buses, while DFS achieved high redundancy and 87.5% generator bus coverage at the expense of significantly higher PMU deployment. These results confirm that SA provides a better trade-off between redundancy, critical bus coverage, and installation cost for practical wide-area monitoring applications. To further evaluate the practical applicability of the obtained PMU placement solutions, the economic and reliability trade-off analysis presented in Table 6 compares the methods in terms of installation cost, redundancy, and fault tolerance capability. Although ILP provides the most economical solution, its redundancy and critical bus protection capability remain limited. DFS achieves the highest reliability and redundancy but requires substantially higher investment and maintenance complexity. In comparison, SA offers a balanced compromise by providing complete generator protection and high redundancy with only a moderate increase in installation cost, making it a more practical solution for reliable wide-area monitoring applications.

To further assess the robustness of the obtained PMU placement solutions under contingency conditions, PMU outage analyses were carried out. Table 7 presents the contingency analysis results under PMU outage condition for different OPP methods. The results show that ILP and RSTS are more vulnerable to PMU failures, where the worst-case PMU outage causes up to 7 buses to become unobservable. In contrast, SA reduces the maximum observability loss to 6 buses, while DFS demonstrates the highest resilience with only 4 buses becoming unobservable under the worst PMU outage scenario. Similarly, the average number of unobserved buses per PMU loss decreases from 3.77 in ILP to 1.24 in DFS, indicating improved redundancy and fault tolerance with increased PMU deployment. The SORI values after the worst PMU outage further highlight the superior robustness of DFS and SA compared to ILP and RSTS. These results confirm that redundancy-based PMU placement improves system robustness against PMU contingencies.

To provide a comprehensive quantitative comparison of the proposed OPP methods, Table 8 summarizes the key performance metrics, including observability redundancy, critical bus coverage, contingency resilience, PMU efficiency, and cost-effectiveness measures obtained for the Indian Utility 62-bus system. ILP provides the most economical solution with the minimum PMU count and highest PMU utilization efficiency; however, its redundancy and critical bus monitoring capability are comparatively lower. DFS achieves the highest SORI, Average DOR, and contingency robustness due to extensive PMU deployment, thereby offering superior fault

tolerance and observability performance. Meanwhile, SA demonstrates a balanced trade-off by achieving complete generator bus coverage and improved redundancy with only a moderate increase in PMU installation cost. The contingency analysis further shows that redundancy-based PMU placement

significantly enhances system resilience against PMU outages. Overall, the results suggest that SA is a practical solution for reliable and cost-effective wide-area monitoring, whereas DFS is more suitable for applications requiring maximum redundancy and reliability.

Table 3. Performance comparison of ILP, DFS, SA, and RSTS algorithm

Methods Bus System	Integer programming (ILP) [46]		Depth First Search (DFS) [6]		Simulated Annealing (SA) [21]		Recursive Spanning Tree Search (RSTS) [24]	
	Number of PMUs (Location)	SORI	Number of PMUs (Location)	SORI	Number of PMUs (Location)	SORI	Number of PMUs (Location)	SORI
IEEE30 BUS	7(2, 4, 10, 12, 19, 24, 27)	35	10 (1, 5, 6, 10, 11, 12, 18, 24, 26, 27)	45	8(1, 5, 6, 10, 12, 18, 24, 27)	39	7 (3, 2, 19, 12, 10, 30, 24)	31
IEEE 57BUS	11 (3, 9, 11, 12, 13, 14, 15, 19, 29, 35, 49)	54	22 (1, 4, 8, 10, 13, 19, 21, 24, 27, 29, 30, 32, 36, 38, 39, 43, 45, 47, 50, 53, 55, 56)	85	14 (1, 4, 9, 15, 19, 27, 30, 32, 38, 39, 41, 50, 52, 54)	59	12 (1, 4, 13, 19, 27, 29, 30, 32, 38, 51, 54, 56)	51
INDIAN UTILITY 62BUS	13 (1, 4, 12, 14, 21, 25, 30, 34, 41, 42, 48, 51, 58)	72	21 (1, 3, 6, 8, 11, 14, 17, 20, 22, 25, 30, 32, 38, 39, 41, 43, 44, 48, 51, 52, 58)	94	15 (1, 8, 14, 20, 22, 25, 30, 32, 38, 39, 41, 43, 49, 51, 58)	69	15 (8, 9, 11, 14, 20, 21, 24, 25, 30, 32, 38, 42, 48, 51, 58)	69
					15 (1, 6, 14, 20, 22, 25, 32, 38, 39, 40, 41, 43, 48, 51, 58)	72	15 (6, 9, 11, 14, 20, 22, 24, 25, 30, 32, 38, 42, 48, 53, 58)	70
					15 (1, 6, 14, 22, 23, 25, 30, 32, 38, 39, 41, 43, 48, 51, 58)	75	15 (6, 9, 11, 14, 20, 22, 24, 25, 30, 32, 38, 42, 48, 53, 58)	70
					16 (1, 3, 14, 17, 20, 25, 30, 32, 37, 38, 41, 43, 44, 48, 51, 58)	79	15 (6, 9, 11, 14, 17, 20, 24, 25, 30, 32, 38, 42, 48, 53, 58)	71
					17 (1, 3, 8, 14, 20, 22, 25, 30, 32, 38, 39, 41, 43, 48, 51, 52, 58)	77	15 (8, 9, 11, 14, 20, 22, 24, 25, 30, 32, 38, 42, 48, 53, 58)	68
					15 (1, 6, 14, 17, 20, 25, 30, 32, 38, 39, 41, 43, 48, 51, 58)	74	15 (6, 9, 10, 12, 14, 22, 24, 25, 30, 32, 38, 42, 48, 51, 57)	69

Table 4. Comprehensive performance analysis of PMU placement methods on the Indian 62-Bus system

Methods	Methods	No of PMUs	PMU locations	Avg. DOR	SORI	Generator Buses observed	DOR of Generator Buses	Total number of Weak Buses observed	DOR of weak buses
ILP	ILP	13	1, 4, 12, 14, 21, 25, 30, 34, 41, 42, 48, 51, 58	1.16	72	5 (1, 14, 25, 51, and 58)	2	1 (30)	1
RSTS	RSTS	15	6, 9, 11, 14, 17, 20, 24, 25, 30, 32, 38, 42, 48, 53, 58	1.15	71	5 (14, 17, 25, 32, and 58)	1	3 (11, 20, and 30)	1
SA	SA	16	1, 3, 14, 17, 20, 25, 30, 32, 37, 38, 41, 43, 48, 51, 58	1.21	79	8 (1, 14, 17, 25, 32, 37, 51, and 58)	1.63	3 (3, 20, and 30)	1
DFS	DFS	21	1, 3, 6, 8, 11, 14, 17, 20, 22, 25, 30, 32, 38, 39, 41, 43, 44, 48, 51, 52, 58	1.52	94	7 (1, 14, 17, 25, 32, 51, and 58)	1.43	3 (3, 20, and 30)	1

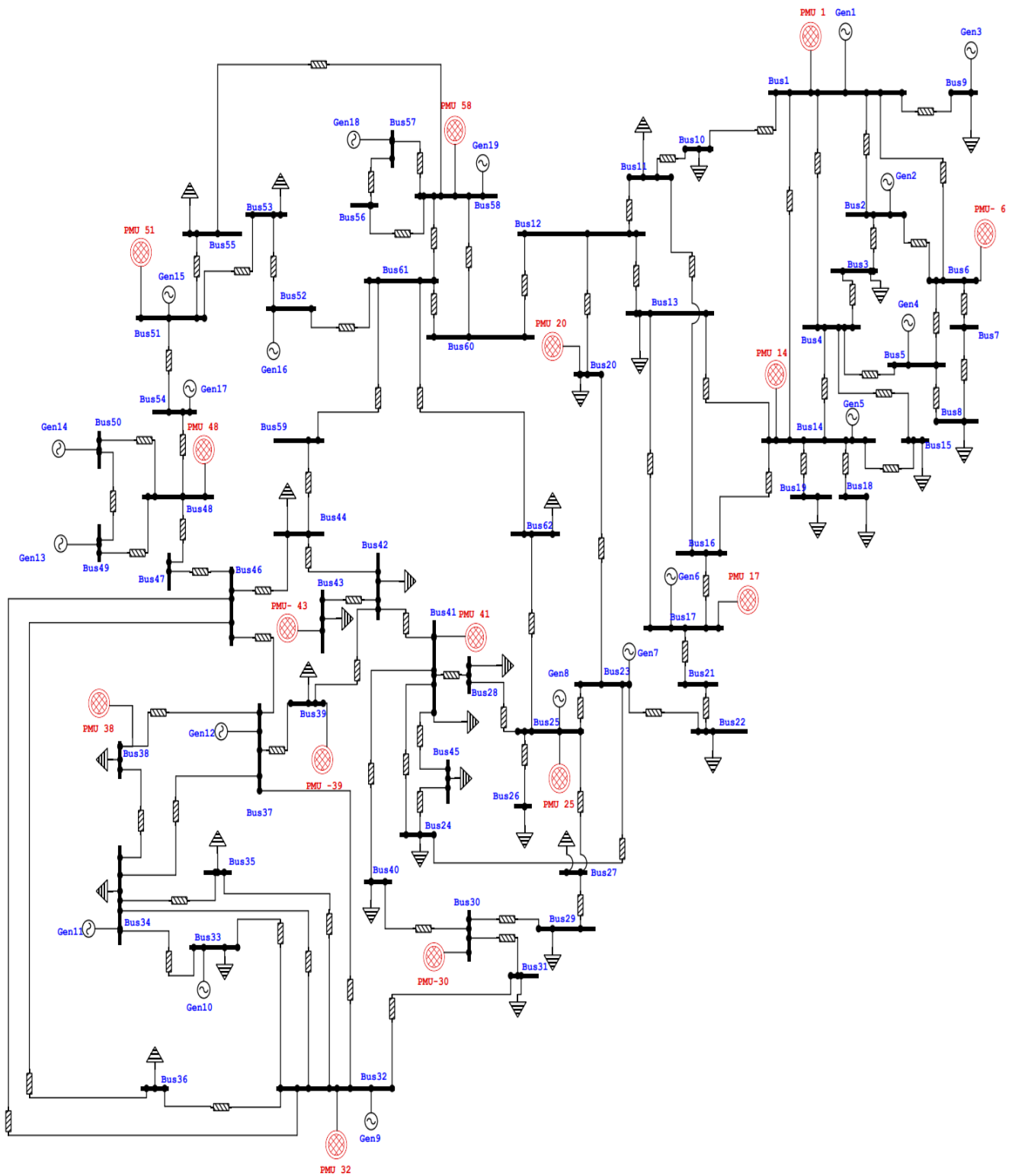


Fig. 5. Single-line diagram of the Indian utility 62 bus system with PMUs.

Table 5. Critical bus coverage analysis

Bus Type	Total Count	ILP Coverage	RSTS Coverage	SA Coverage	DFS Coverage	Best Coverage Method
Generator Buses	8	5 (62.5%)	5 (62.5%)	8 (100%)	7 (87.5%)	SA (Complete)
Weak Buses	3	1 (33.3%)	3 (100%)	3 (100%)	3 (100%)	RSTS /SA/DFS
Total Observable	62	62 (100%)	62 (100%)	62 (100%)	62 (100%)	All methods
Critical Redundancy	-	Low	Moderate	High	Highest	DFS

Table 6. Economic and reliability trade-off analysis

Economic Factor	ILP	RSTS	SA	DFS	Optimal Choice
*Installation Cost	\$650K	\$750K	\$800K	\$1,050K	ILP (Lowest)
Cost per SORI Point	\$9.03K	\$10.56K	\$10.13K	\$11.17K	ILP
Redundancy Investment	Baseline	+\$100K	+\$150K	+\$400K	Context-dependent
Fault Tolerance	Low	Moderate	High	Highest	DFS
Generator Protection	Partial	Partial	Complete	Good	SA
Maintenance Complexity	Low	Moderate	Moderate	High	ILP

*Assuming \$50K per PMU installation cost

Table 7. N-1 Contingency analysis (PMU)

Methods	No of PMUs	SORI baseline	Max unobserved buses (Worst PMU Loss)	Average unobserved buses (per PMU Loss)	SORI after the Worst PMU Loss	Avg. DOR after the Worst PMU Loss
ILP	13	72	7	3.77	69	1.14
RSTS	15	71	7	2.93	67	1.08
SA	16	79	6	2.69	74	1.19
DFS	21	94	4	1.24	86	1.87

*Critical PMU: The PMU whose outage causes at least one bus to lose observability

**Worst PMU: The PMU whose outage causes the maximum number of buses to go unobservable

Table 8. Comparative performance metrics and improvement

Performance Metric	ILP	RSTS	SA	DFS	Best Method	Improvement over ILP
PMU Count	13	15 (+15.4%)	16 (+23.1%)	21 (+61.5%)	ILP	Baseline (most economic)
Average DOR	1.16	1.15 (-0.9%)	1.21 (+4.3%)	1.52 (+31.0%)	DFS	+31.0%
SORI	72	71 (-1.4%)	79 (+9.7%)	94 (+30.6%)	DFS	+30.6%
Generator Coverage	5/8 (62.5%)	5/8 (62.5%)	8/8 (100%)	7/8 (87.5%)	SA	+60% coverage
Avg Generator DOR	2.0	1.0 (-50%)	1.63 (-18.5%)	1.43(-28.5%)	ILP	Baseline
Cost-Effectiveness Ratio*	6.42	5.61	5.98	7.25	DFS	+12.9%
PMU Efficiency**	4.77	4.13	3.88	2.95	ILP	Baseline
Contingency Resilience	Low	Moderate	High	Highest	DFS	Significant improvement

*Cost-Effectiveness Ratio = (SORI × DOR) / PMU Count

**PMU Efficiency = Total Observable Buses (62) / PMU

6. Key Performance Insights and Selection

The comparative analysis of the considered OPP techniques demonstrates that each method provides a different trade-off between installation cost, observability redundancy, and contingency resilience. Therefore, the selection of an appropriate PMU placement strategy should depend on the operational priorities and reliability requirements of the utility.

➤ For Budget-Constrained Utilities:

The ILP configuration is recommended for utilities prioritizing minimum installation cost and reduced PMU deployment. Although the redundancy level is comparatively lower, the method provides complete system observability with the minimum number of PMUs, making it suitable for initial PMU deployment planning and economically constrained applications.

➤ For Reliability-Critical System Applications:

The SA configuration offers a balanced compromise between deployment cost and monitoring reliability. The method achieves higher SORI and DOR values, improved generator bus observability, and stronger resilience under PMU outage contingencies. Therefore, SA is more suitable for real-time monitoring applications where redundancy and fault tolerance are critical operational requirements.

➤ For Highly Secure or Defence-Oriented Applications:

The DFS configuration provides the highest observability redundancy and the strongest contingency resilience due to the larger number of deployed PMUs. Although the installation cost is significantly higher, the improved fault tolerance and

redundancy make this method appropriate for highly critical infrastructures requiring maximum monitoring reliability.

➤ For Standard Utility Monitoring Applications:

The RSTS configuration provides a practical balance between PMU count, redundancy enhancement, and implementation cost. The method maintains satisfactory observability performance while achieving improved robustness compared to minimum-PMU solutions, making it suitable for conventional utility monitoring environments.

➤ For Long-Term Smart Grid Expansion Planning:

Utilities may initially adopt economical ILP-based deployment and progressively transition toward redundancy-oriented configurations such as SA or RSTS as reliability requirements, contingency considerations, and monitoring infrastructure evolve.

Overall, the presented implementation guidelines indicate that optimal PMU placement should not be evaluated solely on minimum PMU count. Observability redundancy, critical bus coverage, and contingency resilience must also be considered to ensure reliable and secure real-time monitoring of modern power systems.

7. Conclusion

This paper presents a comprehensive review and analysis of various OPP techniques, introducing the novel DOR metric for direct observability of voltage-critical buses, alongside SORI, to provide an enhanced assessment of system observability. Unlike existing approaches that primarily focus on achieving complete observability with minimum PMU deployment, the proposed DOR metric provides additional

insight into bus-level redundancy and critical bus protection under contingency conditions. The performance of different OPP methods is evaluated on IEEE 30-bus and IEEE 57-bus systems for validation, and subsequently implemented on the Indian Utility 62-bus system.

The obtained results show that ILP provides the most economical solution with minimum PMU deployment (13 PMUs), whereas stochastic approaches such as SA and RSTS achieve improved redundancy and contingency resilience. Among all methods, the SA-based configuration provided the best overall balance with 16 PMUs, SORI = 79, average DOR = 1.21, complete generator bus observability (100%), and full weak bus coverage. In contrast, DFS achieved the highest redundancy performance with SORI = 94, average DOR = 1.52, and the strongest resilience under PMU outage contingencies, although at significantly higher installation cost. The contingency analysis further confirmed that redundancy-oriented PMU placement significantly improves system robustness compared to minimum-PMU solutions. Overall, the results demonstrate that the selection of an appropriate OPP technique should depend on utility priorities such as cost, redundancy, and monitoring reliability. Future work could extend this redundancy-aware placement framework by integrating dynamic performance metrics such as voltage stability indices, contingency ranking, or communication latency impact, further refining PMU placement strategies with some advanced AI techniques in modern smart grids.

Acknowledgements

The authors would like to express their sincere gratitude to colleagues and others who supported and encouraged them and contributed directly or indirectly to this work.

Author Contributions

Ms. Nisha Sharma: Conceptualization, Methodology, Formal analysis, Investigation, Writing-Original Draft, Writing- review & editing, Visualization. **Dr Priti Prabhakar and Dr. Avneesh Verma:** Validation, review & editing.

Conflict of Interest

There is no conflict of interest regarding the publication of this paper.

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